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Water-Wave Effects on Radio Wave Propagation in the Ocean

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ABSTRACT

A sinusoidal surface profile is used to study by an exact method the effect of water waves on an electromagnetic field propagating downwards from the surface. It is assumed that the magnetic field is directed parallel to the surface corrugations. The results are presented graphically.

Comparisons are made between these results and those obtained using an approximate method of Wait.

Accepted for the Air Force
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I. METHOD OF SOLUTION

The problem under consideration is the evaluation of the electromagnetic field components under the ocean when the field sufficiently far above the ocean is essentially uniform and horizontal. In a Cartesian coordinate system with its z axis pointing vertically, the ocean surface is assumed to be given by the equation

$$z + a \cos \varkappa x = 0. \tag{1}$$

where $\varkappa = 2\pi/L$ and L is the water wavelength. The magnetic field $\underset{\sim}{H}$ far above the surface is assumed to lie in the y-direction, and so the symmetry of the problem forces it to remain so everywhere. Since, in addition, neither the far field nor the surface shape is a function of y, then no field quantity will depend on y.

Now the electromagnetic wavelengths of practical interest are very long compared with the water wavelength. Therefore the quasi-static assumption that displacement currents are negligible can safely be made. Then in the (non-conducting) air, $\underset{\sim}{H}$ satisfies the equation $\nabla \times \underset{\sim}{H} = 0$, $\nabla \cdot \underset{\sim}{H} = 0$. But since $\underset{\sim}{H} = \hat{y} H_y(x, z)$, then the only possible solution in the air is that H_y is a constant everywhere.

In the ocean, $\underset{\sim}{H}$ satisfies the quasi-static equation $(\nabla^2 + k^2)\underset{\sim}{H} = 0$, where $k^2 = i\omega\sigma\mu_o$. (Time dependence is assumed to be as $\exp\{-i\omega t\}$, σ is the ocean conductivity and μ_o is the permeability of free space). Thus since the solution must also be periodic in x, the general solution, if it converges, for H_y is

$$H_y(x, z) = \sum_{n=o}^{\infty} b_n \exp(-ik_n z)\cos n\varkappa x. \qquad (2)$$

(The symmetry of the problem about the plane $x = 0$ excludes the possibility of additional terms in $\sin n\varkappa x$). Here k_n is defined as

$$k_n = \sqrt{k^2 - n^2\varkappa^2} = i\sqrt{n^2\varkappa^2 - k^2} \qquad (3)$$

and the negative sign in the exponent in (2) ensures that each mode decays exponentially as z goes negative.

The corresponding expression for the electric field $\underset{\sim}{E}(x, z)$ is given by $\underset{\sim}{E} = (\nabla \times \underset{\sim}{H})/\sigma = (-\hat{x}\,\partial H_y/\partial z + \hat{z}\,\partial H_y/\partial x)/\sigma$, or

$$\underset{\sim}{E} = \frac{1}{\sigma}\sum_{n=o}^{\infty} b_n\left\{\hat{x}\, ik_n \cos n\varkappa x - \hat{z}\, n\varkappa \sin n\varkappa x\right\}\exp(-ik_n z) \qquad (4)$$

Solving the problem involves finding the values of the unknown b_n coefficients in (2) and (4) by means of the boundary condition that H_y is continuous at the surface. Thus, assuming the uniform field above the surface is normalized to unity, one can formally write, using (1) and (2),

$$\sum_{n=o}^{\infty} b_n \exp\left\{ik_n a \cos\varkappa x\right\}\cos n\varkappa x = 1. \qquad (5)$$

By truncating the series on the left to N terms and enforcing the equality at N points in the range $0 \le x \le L$ one can obtain N equations in the first N unknown b_n coefficients. These equations can then be solved by the usual methods. Unfortunately, to obtain (5), one must make the Rayleigh assumption, which is that the downward-going wave expansion (2)

is valid not only for $z < -a$ but also in the strip $-a \leq z \leq a$. It can be shown [1, 2] that if the normalized wave height a/L is greater than 0.713 then (2) is invalid there and (5) diverges. Thus the b_n found by truncating the series in (5) to N terms will not in general converge to their correct values as N is increased.

However, by interpreting (5) in a more general sense, one can still use it as a basis for finding the b_n even though the series diverges [3, 4]. That is, one regards the left side of (5) as a generalized function which can be equated to the actual field on the surface only indirectly via a complete set of sufficiently smooth test functions. In this sense the downward going wave expansion will remain valid on the surface for a much larger range of a than in the conventional point-by-point sense.

Thus although (5) may be incorrect as it stands, the result of multiplying each side by $(\varkappa/2\pi)\cos m\varkappa x$ and integrating with respect to x over the range 0 to $2\pi/\varkappa$ is the equation

$$\sum_{n=o}^{\infty} A_{mn} b_n = \delta_{mo} \tag{6}$$

where $\delta_{mo} = 1$ if $m = 0$ and is zero otherwise, and A_{mn} is

$$A_{mn} = \frac{\varkappa}{2\pi}\int_{o}^{2\pi/\varkappa} \exp(i\, k_n a \cos \varkappa x) \cos n\varkappa x \cos m\varkappa x \; dx$$

$$= \left\{ i^{n+m} J_{n+m}(k_n a) + i^{n-m} J_{n-m}(k_n a) \right\} / 2. \tag{7}$$

Here the chosen test functions are the set $(\varkappa/2\pi)\cos m\varkappa x$, $(\varkappa/2\pi)\sin m\varkappa x$, of which only the former are necessary for representing the even function of x which is H_y, and the series in (6) converges over a much larger range

of a than does the series in (5). In (7), $J_{n+m}(k_n a)$ is the Bessel function of the first kind [5] of order n+m and argument $k_n a$.

The evaluation of the b_n is now carried out in a straightforward way by truncating the series and solving the resulting finite set of equations.

To obtain the actual field quantities in the ocean from the b_n is a simple matter in the region $z < -a$ where the downward going wave expansion is known to converge. One simply substitutes the numerical values of the b_n into (2) for H_y and into (4) for $\underset{\sim}{E}$. However, in the region $-a \leq z \leq a$, the series in (2) and (4) will not in general converge, and so cannot be used directly.

A general method which is applicable for evaluating the field quantities anywhere beneath the surface is to take that function of x which is the field quantity along the line $z = z_d(x) = -a \cos \varkappa x - d$ and expand it in the test function set. In this case, for H_y for example, an even function of x, the expansion is

$$H_y(x, z_d) = \sum_{n=o}^{\infty} c_n(d) \cos n\varkappa x. \tag{8}$$

But the generalized function for this same $H_y(x, z_d)$ is given by (2) with z replaced by $z_d(x)$. Thus the $c_n(d)$ can be evaluated by multiplying the right sides of both (2) and (8) by $(\varkappa/2\pi) \cos m\varkappa x$ and integrating from 0 to $2\pi/\varkappa$. The result is

$$c_n(d) = \epsilon_n \sum_{m=o}^{\infty} A_{nm} \exp(ik_m d) b_m, \tag{9}$$

where $\epsilon_n = 1$ if $n = 0$ and is 2 otherwise, and both the b_m and the A_{nm} [defined by (7)] are already known.

Thus by using (8) and (9) one evaluates H_y on contours which are the same shape as the surface contour but at an arbitrary depth d below it. (When $d = 0$ the contour lies at the surface, on which $H_y = 1$. Thus from (8), the $c_n(0)$ are given by $c_n(0) = \delta_{no}$ and so (9) reduces to (6), the equation determining the b_n.)

Similarly, by expressing $\underset{\sim}{E}$ on the shifted surface-shaped contour as

$$\underset{\sim}{E} = \hat{x} \sum_{n=o}^{\infty} a_n(d) \cos n\varkappa x - \hat{z} \sum_{n=1}^{\infty} s_n(d) \sin n\varkappa x$$

one finds

$$a_n(d) = \epsilon_n \sum_{m=o}^{\infty} A_{nm} \exp(ik_m d)\, ik_m b_m/\sigma$$

and

$$s_n(d) = 2 \sum_{m=1}^{\infty} B_{nm} \exp(ik_m d)\, m\varkappa b_m/\sigma$$

where A_{nm} is defined by (7) and

$$B_{mn} = [i^{n-m} J_{n-m}(k_n a) - i^{n+m} J_{n+m}(k_n a)]/2.$$

It should be noted that although the method described above can be expected to converge numerically to the exact values for the coefficients b_m, it will not in general be the case that $|b_m|$ goes to zero as m grows indefinitely, for the series in (2) does not in general converge when $z = 0$. Since (2) does converge for $z \leq -a$, however, one should find that the modified coefficients $b_m \exp(i k_m a)$ do go to zero in absolute value as m grows indefinitely.

II. RESULTS

Approximations of the b_m coefficients were obtained by inverting the following truncated version of (6)

$$\sum_{n=o}^{N} A_{mn} b_n = \delta_{mo} (m = 0, 1, \ldots, N) \qquad (10)$$

for various values of N and of the problem parameters δ/L and a/L. ($\varkappa = 2\pi/L$ and $k = (1+i)/\delta$, where $\delta = \sqrt{2/\omega \sigma \mu_o}$ is the skin depth in the water.) As a check on the accuracy of the inversion, the b_n values found from it were substituted back in the left side of (10) and the result compared with δ_{mo}. As a check on the complete approximation, the sum

$$S = \sum b_n \exp(ik_n a)$$

was computed using the approximate b_m and compared with the correct value, given by (2) with $z = -a$ and $x = 0$, of unity. All computations were performed on the IBM 360 computer using single precision.

It was found that the method converged, as N increased, to give stable values for the b_m when a/L was about 0.3 or less. The rapidity of the convergence was greatest for the smaller values of a/L. The effect of δ/L on the convergence was less marked, but was in the direction that convergence was less rapid for the smaller δ/L values. A typical result is that with $\delta/L = 1$, $a/L = 0.15$ and $N = 9$, the value of S-1 was found to be, in absolute value, less than 10^{-3}.

When values of a/L larger than 0.3 were used, the convergence was slow and the b_m did not settle to stable values. However, the inversion check showed that this behavior was always accompanied by poor inversion accuracy. This indicates that the set of equations (10) becomes poorly conditioned when a/L exceeds 0.3, and then single precision is insufficient to solve them accurately.

Some specific results for the underwater field components are given in Figs. 1 and 2. In Fig. 1, the horizontal field components H_y and E_x in the plane $z = -a$ are plotted as a function of x in magnitude and phase over a half-period $(0 \leq x \leq L/2)$ for various values of δ/L. The wave height parameter a/L is constant at 0.15 (giving a maximum wave slope of about 1) for all curves, and the field quantities are normalized with respect to the fields that would exist at depth a beneath a plane surface.

The points plotted in Fig. 1 are the result of using Wait's [6] approximation to calculate the same field quantities. This method assumes that for sufficiently shallow and long water waves, the field propagates down to the level $z = -a$ essentially as a plane wave. Thus it would be expected to give good results for small δ/L and small a/L. The extent to which the approximation deteriorates when a/L is 0.15 is indicated by the closeness of the points to the continuous lines. (In Wait's approximation, the normalized quantities H_y and E_x are identical, so that only one curve of points exists for amplitude and one for phase.)

At a sufficiently great depth, only the first terms in the downward-going wave expansions (2) and (4) remain significant. Under this condition, the ratio of the actual field to the field that would exist at the same depth beneath a plane surface is simply $b_o/1$ for both H_y and E_x, since $b_o = 1$ when $a/L = 0$. It was found that $|b_o|$ is always greater than or equal to unity, and so the amplitude departure can be represented unambiguously by the quantity $|b_o - 1|$. The phase departure is just the phase of b_o. These quantities are plotted in Fig. 2 as a function of a/L for various values of δ/L.

Since b_o also equals the ratio of the average horizontal field to the field at the same depth below a plane surface, Fig. 2 also shows the way the average horizontal field components depend on a/L and δ/L for any $z \leq a$.

III. CONCLUSIONS

The mathematical technique described here (a generalized-function interpretation of the Rayleigh assumption) appears to be accurate and to converge rapidly for values of a/L for which the conventional interpretation of the Rayleigh assumption is known to be invalid. Using this technique, one can evaluate the electromagnetic field at any point beneath the surface for all water wave heights and lengths of practical interest.

REFERENCES

1. Petit, R. and Cadilhac, M., "Sur la diffraction d'une onde plane par une réseau infiniment conducteur", CR Acad. Sci., 1966, 262B, pp. 468-71.

2. Millar, R. F., "On the Rayleigh Assumption in Scattering by a Periodic Surface", Proc. Camb. Phil. Soc., 1969, 65, pp. 773-91.

3. Burrows, M. L., "Equivalence of the Rayleigh Solution and the Extended-Boundary-Condition Solution for Scattering Problems", Electron. Lett., 1969, 5, pp. 277-8.

4. Burrows, M. L., "Example of the Generalized-Function Validity of the Rayleigh Hypothesis", Submitted to Electron. Lett.

5. Jahnke, E. and Emde, F., Tables of Functions (Dover, 1945), p. 149.

6. Wait, J. R., "The Calculation of the Field in a Homogeneous Conductor with a Wavy Surface", Proc. I. R. E., 1959, pp. 1155-6.

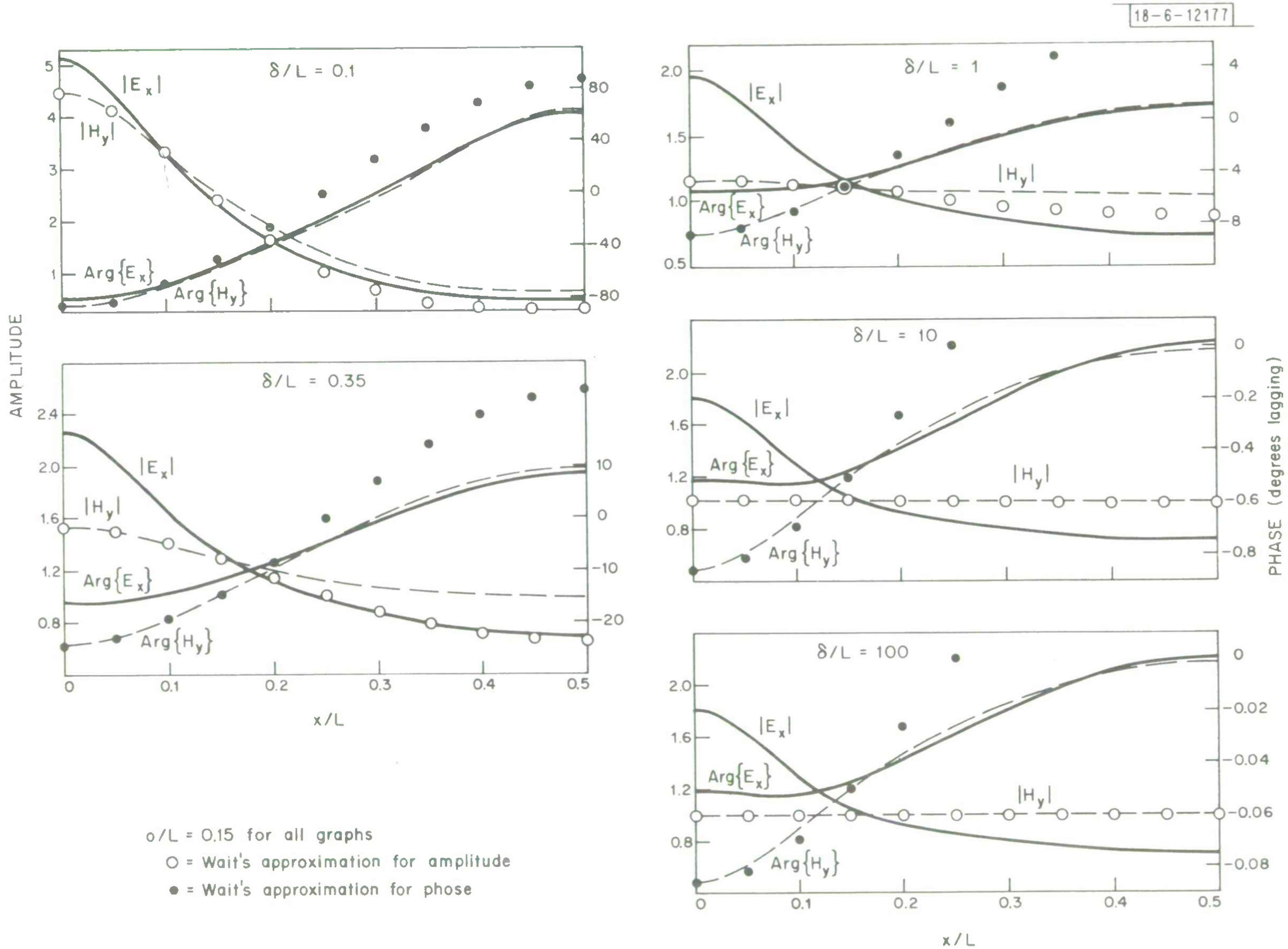

Fig. 1. The amplitude and phase of the horizontal field components E_x and H_y in the plane of the wave troughs as a function of x/L for a/L = 0.15 and various values of δ/L. The fields are normalized with respect to the fields at the same depth (z = -a) beneath a plane surface. The plotted points were calculated using Wait's approximation.

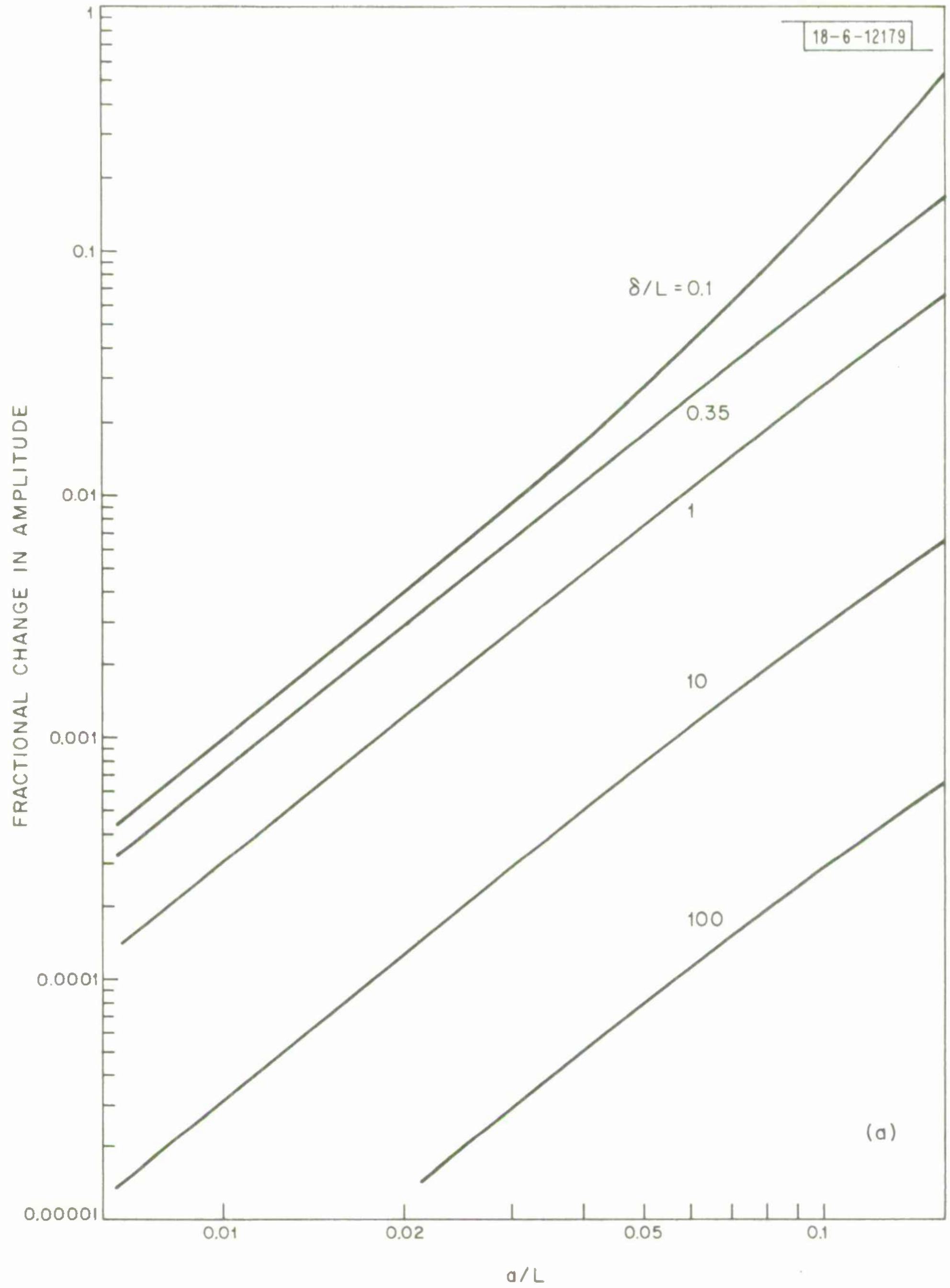

Fig. 2. The amplitude and phase departures of the horizontal field at "great" depth (or of the average horizontal field at any depth $z \leq -a$) from those of the field at the same depth below a plane surface.

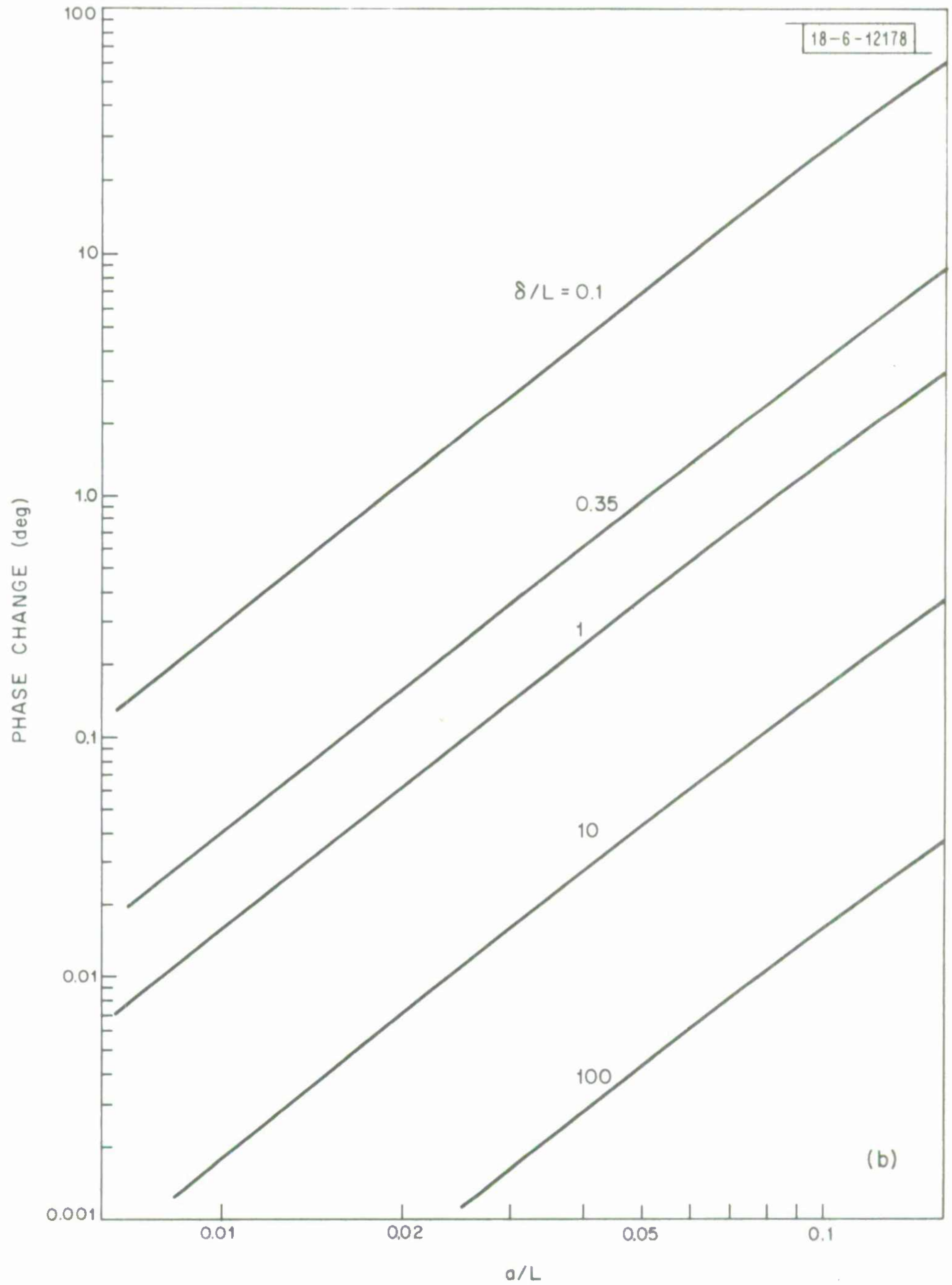

Fig. 2. Continued.

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13. ABSTRACT

A sinusoidal surface profile is used to study by an exact method the effect of water waves on an electromagnetic field propagating downwards from the surface. It is assumed that the magnetic field is directed parallel to the surface corrugations. The results are presented graphically.

Comparisons are made between these results and those obtained using an approximate method of Wait.

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